Two and Three-Dimensional Numerical Analysis of the Interaction Between Open Pit Slope Stability and Remnant Underground Voids

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ABSTRACT

Recent advances in comminution and heap leach technology have resulted in the development of many large-scale, low-grade open pit mines that coincide with areas of historical underground workings. Two and three-dimensional numerical modeling has been conducted to assess the interaction between open pit slope stability and remnant underground voids. Most design analyses for open-pit slopes assume a two-dimensional problem geometry. This is not the condition encountered in practice — particularly when remnant underground voids interact with a large-scale open pit slope. Three-dimensional numerical models have been shown to provide a more realistic representation of the interaction of between overall open pit slope stability and remnant underground voids.

INTRODUCTION

Large-scale open pit mining of low-grade mineral deposits has become common practice due to recent advances in comminution technology and the economic and environmental viability of the heap leach mineral extraction process. Many of these deposits are readily identified by the existence of historic underground workings of associated high-grade deposits, which precludes the need for greenfield mineral exploration. However, the interaction between open pit slopes and remnant underground voids has the potential to cause significant slope stability issues.

Depending on the geometry and location of the underground voids, the interaction with slope stability may cause minor bench-scale instability, multi-bench instability or overall slope instability, as illustrated in Figure 1.

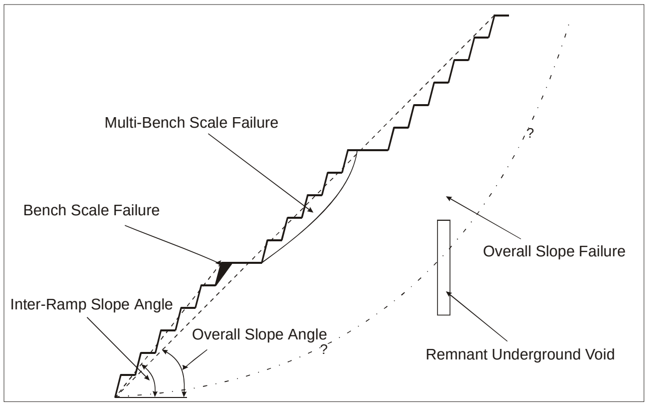

Figure 1. Potential interaction between remnant underground voids and overall slope stability.

OPEN PIT SLOPE INSTABILITY INDUCED BY REMNANT UNDERGROUND VOIDS

The failure mechanisms of open pit coal mines due to the presence of underground workings were studied by Walton and Taylor (1977). The authors report that failure of open-pit slopes was induced by the progression of yielding from around the underground voids.

Significant open pit slope stability concerns were reported by Watters, Finn and Coulthard (1989) and Watters Rehwoldt and Coulthard (1990) at a gold mine in Nevada, USA, due to the interaction of disused underground mine workings. The stability issues reported range from tension cracks, individual bench failures and unraveling of benches in pit slope areas (51^{o} overall angle) intersected by the remnant voids. The underground voids were reported to be primarily subvertical and striking parallel to the slope face. The width and height of the voids were typically 5 and 50 m, respectively, while the strike length of the voids was not reported. At the time of reporting, the overall slope stability had not been affected.

Loubser (1994) reports of multi-bench scale (~100 m) instability due to the presence of a narrow (3-5 m) subvertical remnant underground stope located approximately 50 m behind a 51^{o} overall slope at Kalgoorlie Consolidated Gold Mines Pty. Ltd.'s Super Pit Mine in Kalgoorlie, Western Australia. The strike of the void was reported to be parallel to the slope face with continuous extent. The slope instability subsequently was stabilized by stepping (horizontally) into the pit to form a buttress at the toe of the slope, resulting in the temporary loss of some ore.

PRACTICAL OPEN PIT SLOPE STABILITY ANALYSIS

Knowledge of the potential failure mechanisms of an open-pit slope is required prior to selecting the appropriate method of stability analysis. Where the failure mechanism is structurally controlled, or the failure surface is clearly defined, simple limit-equilibrium methods of analysis may be sufficient for design. Zettler et al. (1999) suggest that the method of slices is the most widely used method in geomechanics to evaluate a factor of safety, by calculating the limiting equilibrium. This method assumes a two-dimensional problem geometry (i.e., a unit slice through an infinitely long slope, under plane-strain conditions) where only plastic failure along a pre-defined discrete slip surface is considered. A number of assumed failure surfaces are tested, and the one giving the lowest factor of safety is chosen. Equilibrium is only satisfied on an idealized set of surfaces.

Unlike bench-scale failures and structurally controlled slope failures, the failure mechanisms of large overall open pit slopes are poorly understood, primarily as there have been few cases of large-scale (400 – 700 m) slopes failures. For such complex failure mechanisms, which involve the deformation behavior of large volumes of rock, multiple failure surfaces, or the interaction of underground voids, it is not appropriate to use a limiting equilibrium method of analysis.

An alternative method of analysis is to apply finite-element or finite-difference numerical methods to the problem. FLAC (Fast Lagrangian Analysis of Continua, Itasca Consulting Group, Inc., 2000) is a two-dimensional explicit finite-difference program developed especially for geomechanical analysis. The code is formulated to study continuum problems whereby a full solution of the coupled stress/displacement, equilibrium and constitutive equations is provided.

The advantages of a finite difference solution over a limit equilibrium solution include the following.

- Any failure mode develops naturally; there is no need to specify a pre-defined failure surface.
- No artificial parameters, such as inter-slice force angles, need to be given as input.
- Multiple failure surface or complex internal yielding evolves naturally.
- Structural interaction (rock bolts) is modeled realistically as fully coupled deforming elements, not simply as equivalent forces.
- The solution consists of mechanisms that are feasible kinematically. Limit equilibrium methods only consider forces, not kinematics.

Until recently, three-dimensional analyses were relatively uncommon in open pit slope analysis. However, recent advances in personal computing have permitted three-dimensional analyses to be performed routinely. FLAC^{3D} (Fast Lagrangian Analysis of Continua in 3 Dimensions, Itasca Consulting Group, Inc., 2002) is the three-dimensional equivalent of FLAC.

Lorig and Varona (2000) suggest that three-dimensional analyses are required/recommended if:

- The direction of principal geologic structure does not strike within 20^o to 30^o of the strike of the slope,
- The directions of principal stresses are thought to be not parallel or not perpendicular to the slope,
- Distribution of geomechanical units vary along the strike of the slope, or
- The slope geometry in plan cannot be represented by two-dimensional (i.e., axisymmetric or plain-strain analysis.

Calculating Slope Factor of Safety with Numerical Methods

The traditional definition of the factor of safety for slope stability analysis is to calculate the factor of safety with respect to the soil/rock shear strength. The factor of safety of a slope can be computed with a finite element or finite difference code by reducing the rock shear strength in stages until the slope fails. The resulting factor of safety is the ratio of the actual shear strength to the reduced shear strength at failure. This method is called the shear strength reduction technique and is discussed in detail by Dawson and Roth (1999), Zettler et al. (1999) and Dawson et al. (1999). Dawson and Roth (1999) show that shear-reduction factors of safety are generally within a few percent of factors of safety computed via limit equilibrium solutions. To perform slope-stability analysis with the shear-reduction technique, actual shear-strength properties, cohesion (c) and friction angle (ϕ), are reduced according to the following equations:

$$c^{\text{trial}} = (1/F)\ c$$

$$\phi^{\text{trial}} = \arctan\{(1/F)\ \tan\phi\}$$

where: c = cohesion,
ϕ = friction angle, and
F = factor of safety.

The shear strength reduction technique has a number of advantages over conventional limit-equilibrium stability analyses based on the method of slices. First, it is not necessary to specify the shape of the failure surface in advance, as the critical failure surface (or multiple failure surfaces) evolves naturally; second, finite element and finite difference codes automatically satisfy translational and rotational equilibrium, unlike many limit equilibrium solutions.

Open pit Slope Stability Modeling Methodology

It is important to understand the limitations and assumptions inherent to a particular numerical method selected. The uncertainty in numerical modeling generally arises from the poor knowledge of the material behavior of rock masses. Owing to the complexity of a rock mass and the uncertainty of its behavior, it is important that any numerical model that is used for slope design be subject to detailed calibration, whereby the model predictions are compared to the observed and measured response of the slope.

A numerical method such as the finite element or finite difference method is required to analyze open pit slope geometries in which underground voids exist behind the slope.

In order to investigate the interaction between open pit slope stability and the presence of remnant underground voids, a conceptual, two-dimensional, large-scale open pit slope was analyzed with FLAC. The model geometry used is illustrated in Figure 2, while the material properties assumed are presented in Table 1. The rock mass was represented as a homogeneous isotropic material with the linear elastic, perfectly plastic Mohr-Coulomb constitutive model. The influence of groundwater was ignored throughout the analyses, due to the presence of underground voids.

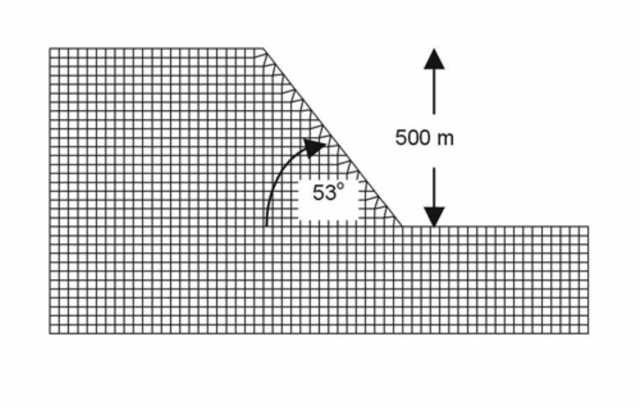

Figure 2. Model geometry

Density (kg/m^3)	2900
Young's Modulus (GPa)	15
Poisson's ratio	0.2
Cohesion (MPa)	1
Friction angle (deg.)	40
Tensile strength (MPa)	0.1
Dilation Angle (deg.)	10

Table 1. Conceptual model material properties

Figure 3 presents the failure surface (delineated by the shear strain-rate contours and velocity vectors) that develops at the last non-equilibrium state of the shear-reduction routine. The factor of safety of the slope is 1.54. The circular failure surface is observed to propagate from the toe of the slope. This is consistent with the mechanism described by Sjöberg, (1999) and Call et al. (2000), whereby accumulation of shear strain starts at the toe of the slope, where the highest shear stresses are found (due to the removal of confining stress during mining), and progresses upward.

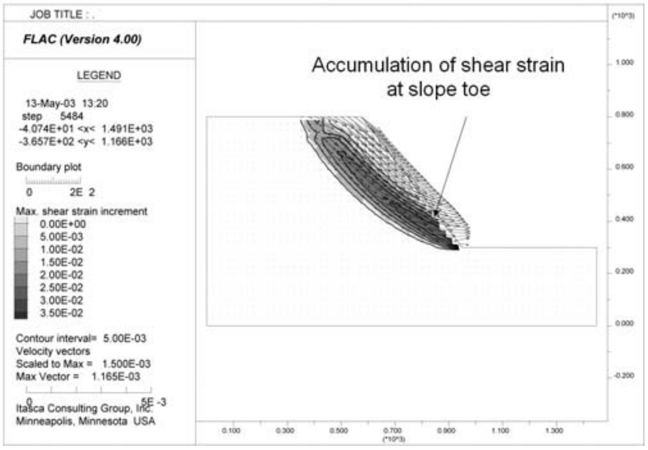

Figure 3. Failure surface of conceptual model

Two-Dimensional Numerical Analysis of Remnant Underground Void Interaction With Slope Stability

Two conceptual underground void geometries were incorporated into the model to investigate the interaction of underground voids with the open pit slope stability: a single open stope, and a single longhole open stope.

Single Open-Stope Analysis

Figure 4 illustrates the geometry used to represent the single open-stope geometry. Although many remnant underground voids are backfilled, the underground void was represented as an open void.

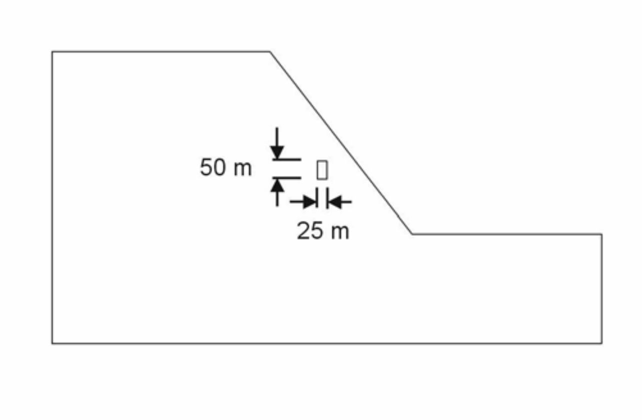

Figure 4. Single open-stope geometry

A total of 65 separate analyses were conducted with the single open stope positioned at various locations behind the slope. Figure 5 presents the failure surface that develops with the single open stope located 125 m above the pit floor and 75 m behind the pit face. The factor of safety of the slope is 1.46, reduced from 1.54 due to the underground void. The accumulation of shear strain is observed to propagate from the underground void. This is consistent with Walton and Taylor's (1977) observation of open pit slope failure induced by progressive yielding around underground voids.

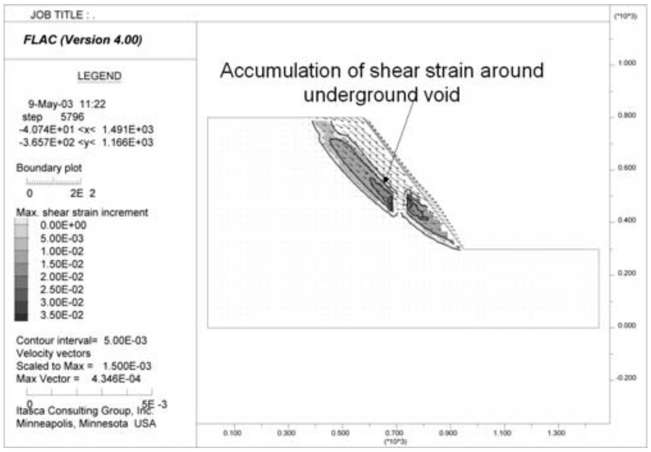

Figure 5. Failure surface of an open pit slope with a single open stope located 125 m above pit floor and 75 m behind face

The factor of safety calculated for each individual stope location is presented in Figure 6. The contour plot illustrates the interaction of a single open stope with the overall open pit slope stability. As the stope is located closer to the toe of the slope, the factor of safety is reduced. The minimum factor of safety is 1.36.

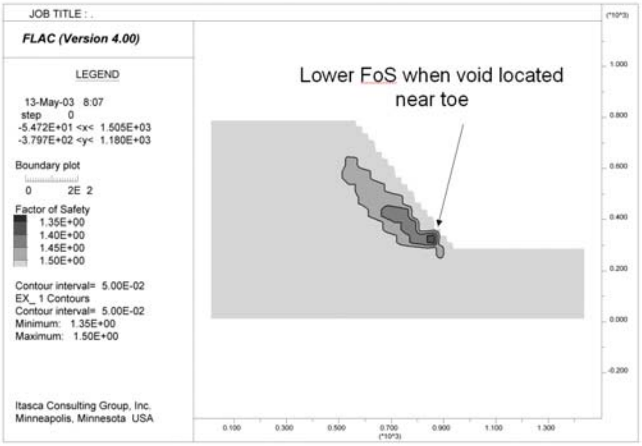

Figure 6. Factor of safety based upon single open stope location

Single Longhole Open-Stope Analysis

Figure 7 illustrates the geometry used to represent the single longhole open-stope geometry.

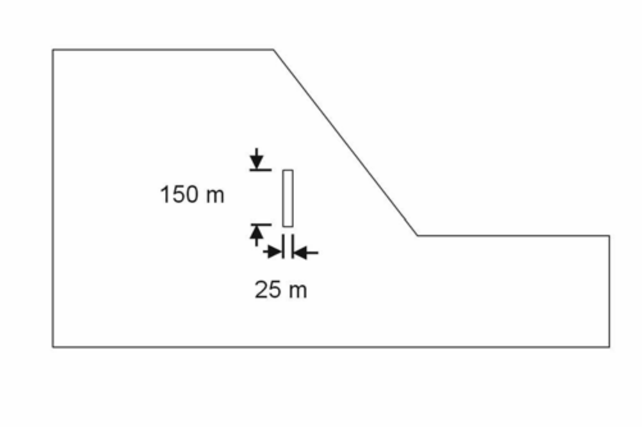

Figure 8. Single longhole open-stope geometry

A total of 42 separate analyses were conducted with the single longhole open stope positioned at various locations behind the open pit slope. Figure 9 presents the failure surface that develops with the center of a single longhole open stope located 62.5 m above the pit floor and 125 m behind the pit face. The factor of safety of the slope is 1.28, reduced from 1.54 due to the underground void. The accumulation of shear strain is observed to propagate from the underground void and form multiple failure surfaces.

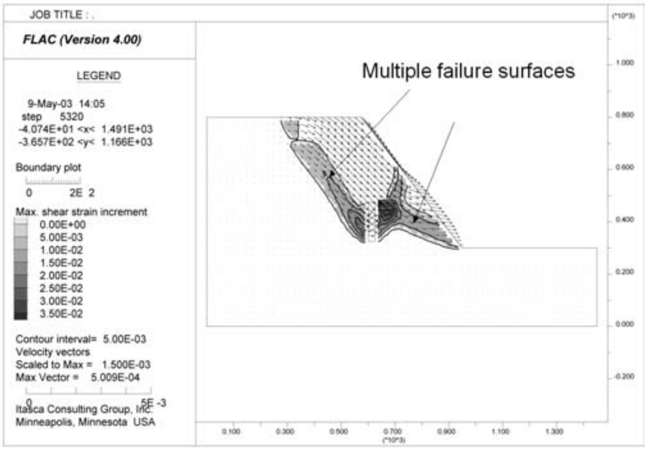

Figure 9. Failure surface of an open pit slope with a single longhole open stope located 62.5 m above pit floor and 125 m behind the face

The factor of safety calculated for each individual stope location is presented in Figure 10. The contour plot illustrates the interaction of a single longhole open stope with the overall open pit slope stability. Due to the geometry of the longhole open stope, the void has a greater influence on the overall slope stability, as evidenced by the development of multiple failure surfaces in Figure 9.

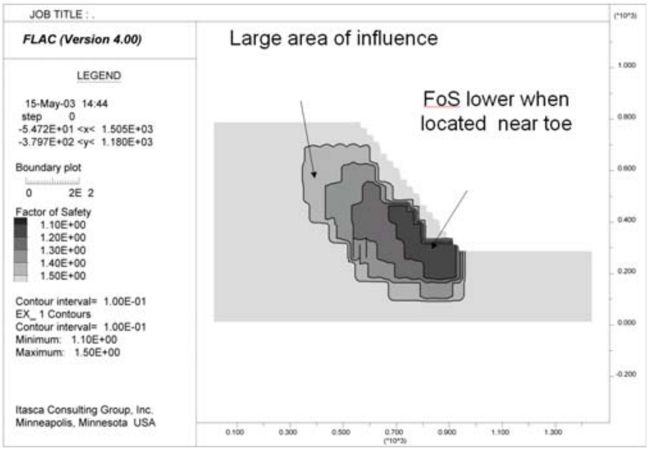

Figure 10. Factor of safety based upon a single longhole open-stope location

Discussion of Two-Dimensional Numerical Analyses

The two-dimensional numerical modeling results indicate that the effect on stability of a single open stope behind a large-scale open pit slope is relatively minor. However, the effect of a longhole open stope with a lower width to height ratio is more significant, reducing the factor of safety of overall slope stability from 1.54 to below 1.20 when located in the toe area. The low width to height aspect ratio of the longhole open stope causes a complex failure mechanism with multiple failure surfaces.

The two-dimensional analyses conducted with FLAC assume plain-strain conditions (a unit slice through an infinitely long slope and infinitely long underground void). This is not the condition encountered in practice and clearly results in a conservative estimation of the slope factor of safety.

Three-Dimensional Numerical Analysis of Remnant Underground Void Interaction With Slope Stability

A series of three-dimensional analyses was conducted with FLAC^{3D} to investigate the effect of underground void length. The two-dimensional conceptual open pit slope was extended to a 500 m long slope with roller boundaries applied to the slope ends to simulate symmetry. The same longhole open-stope width and height dimensions as the two-dimensional analyses were used with length dimensions of 100, 200, 300, and 400 m. The single longhole open stope was located 62.5 m above the pit floor and 125 m behind the pit face.

Figure 11 illustrates the conceptual slope with a 300 m long longhole open stope. The symmetry slope boundaries simulate the behavior of a slope with consecutive 300 m long voids spaced 200 m apart. With no underground voids, the slope factor of safety was the same as the two-dimensional FLAC analysis, 1.54.

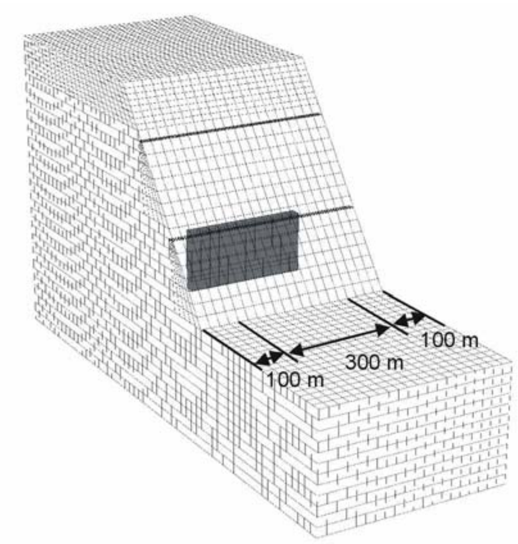

Figure 11. Model geometry with a 300 m long longhole open stope.

Figure 12 illustrates the three-dimensional failure surface of the slope with consecutive 300 m long voids spaced 200 m apart. The complex failure mechanism with multiple failure surfaces is present only in the center of the slope coincident with the void. The factor of safety calculated was 1.47.

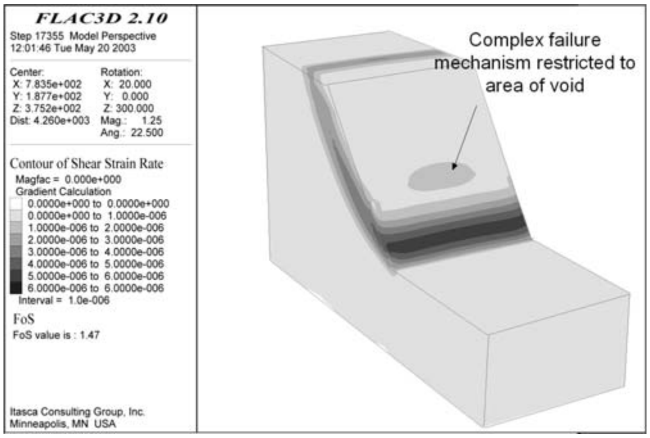

Figure 12. Three-dimensional failure surface of the slope with consecutive 300 m long voids spaced 200 m apart

Figure 13 illustrates the interaction of the longhole open-stope length with the overall slope stability. As the void length increases from 200 m (spaced at 300 m) to 400 m (spaced at 100m), the factor of safety reduces from 1.54 to 1.24.

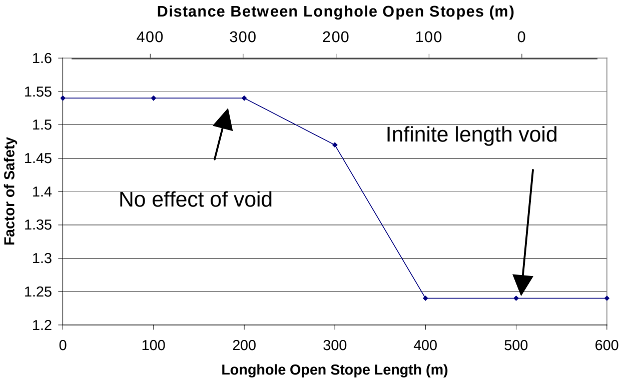

Figure 13. Interaction of the longhole open-stope length with the overall slope stability

Discussion of Three-Dimensional Numerical Analyses

The results of the three-dimensional analyses illustrate that the interaction of remnant underground voids with the overall open pit slope stability is sensitive to the three-dimensional geometry of the void, highlighting the over conservative nature of two-dimensional, plane-strain analyses for such problem geometries.

PRACTICAL EXAMPLES OF REMNANT UNDERGROUND VOID INTERACTION WITH SLOPE STABILITY

Cresson Mine, USA

Many large open stopes remaining from underground mining in the 1900s are located just below the central region of the future pit floor at Cripple Creek & Victor Gold Mining Company's Cresson Mine in Colorado, USA. The effect of the presence of the stopes on overall slope stability was investigated for the proposed West Wall using FLAC^{3D} (Pierce et al., 1999 and Pierce, Brandshaug and Ward, 2000).

The problem geometry shown in Figure 14 involves three different materials: Breccia, Phonolite and a Lamprophyre Breccia intrusion called the Cresson Pipe. The estimated rock-mass strengths used for these units in the FLAC^{3D} analysis are presented in Table 2.

Material	**Cohesion (MPa)**	**Friction (°)**
Breccia	0.78	39
Phonolite	0.58	33
Cresson Pipe	0.14	27

Table 2. FLAC^{3D} model material properties

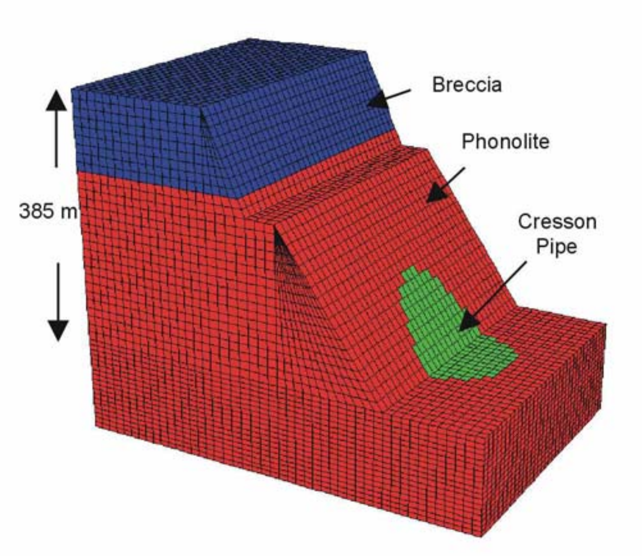

Figure 14. Model geometry

The total proposed wall height shown in Figure 14 is 385 m; the upper part is 145 m high with a slope angle of 64°, while the lower part is 240 m high with a slope angle of 55°. The inter-ramp slope geometry has been simplified by removing individual benches. The Cresson Pipe is assumed to exist within a vertical cylinder with a diameter equal to 175 m. The remnant stopes extend around the east and west peripheries of the Cresson Pipe. The stopes are approximately 8 m wide and 120 m tall. The tops of the stopes correspond to the elevation of the toe of the slope. Both stopes lie approximately 80 meters away from the toe, as shown in Figure 15.

The model was run to obtain factors of safety against failure with and without the stopes present. In both cases, the slope is stable with a factor-of-safety of approximately 1.15, suggesting that the remnant stopes will not affect overall slope stability. This is due to the fact that the stopes lie below the base of the slope and at large distances from the toe of the slope. While the stopes were found to have no negative effect on overall stability of the future West Wall, other stopes that have been encountered in close proximity to the pit wall have resulted in local bench-scale stability problems.

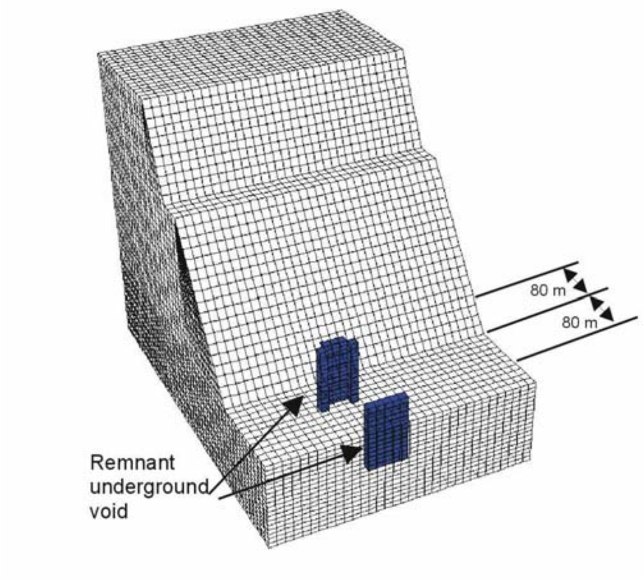

Figure 15. Location of Remnant Stopes in the FLAC^{3D} Model Relative to the West Wall

Large-Scale Open Pit, Chile

The following paragraphs illustrate a more comprehensive analysis of the potential effects of remnant underground openings. Both two and three-dimensional analyses were performed at a large open pit mine in Chile. The total wall height is approximately 380 m, while the overall slope angle is 56^{o}. Remnant underground stopes approximately 125 m wide and 50 m tall are present behind the pit slope. The stopes are located approximately 50 m above the pit floor and approximately 150 m behind the slope face. The rock mass material properties are shown in the Table 3.

Material	Cohesion (MPa)	Friction (^{o})
Andesite	0.12	31
Dacite	0.15	27
Dacite Porphyry	0.07	30

Table 3. Rock-mass material properties.

Two-Dimensional Analysis

Figure 16 shows the resultant failure surface from safety factor analysis using FLAC. The failure mode is strongly affected by the presence of the remnant underground opening, which is assumed to be infinite in extent, and perpendicular to the two-dimensional analysis plane.

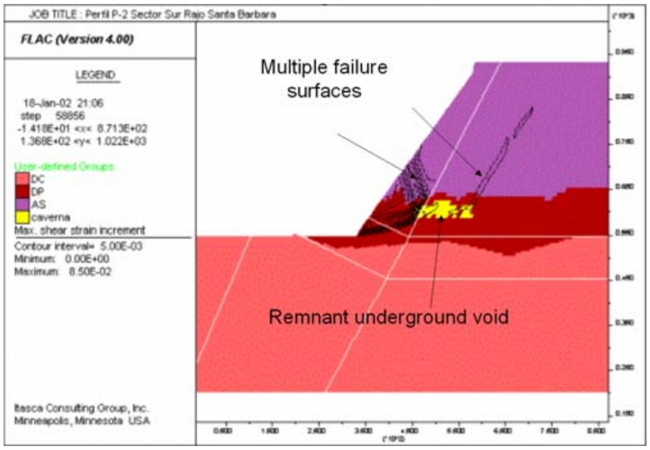

Figure 16. Failure mode determined using FLAC (FS = 1.30)

Three-Dimensional Analysis

Three-dimensional analysis of the problem geometry was conducted with the stress analysis program 3DEC (3-Dimensional Distinct Element Code, Itasca Consulting Group, Inc., 2003). 3DEC treats the rock mass in much the same way as FLAC^{3D}, using an elastic-plastic stress-strain relation to describe the continuum behavior. In addition, faults are represented explicitly as planar failure surfaces that allow slip, rotation, and/or separation of adjacent deformable blocks of material. Figure 17 illustrates the 3DEC model geometry with zones of different material. Material models can be imported easily from geology block models, while the pit geometry can be imported easily from survey models or pit plans.

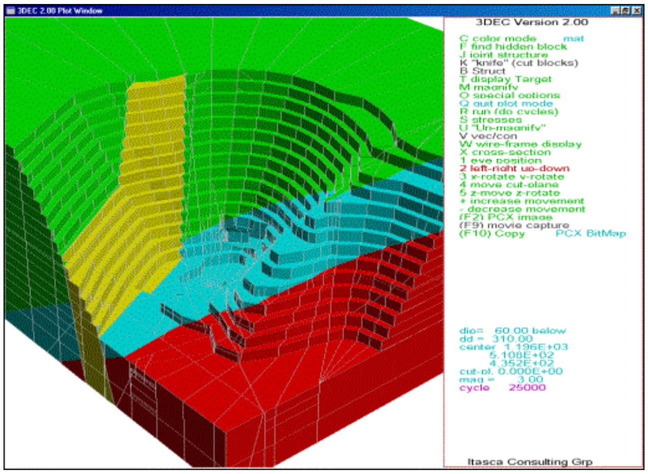

Figure 17. Materials included in 3DEC model.

Figure 18 shows a vertical section through the 3DEC model that is the same as the section shown in Figure 16. The 3DEC model shows some small areas of plastic behavior around the remnant underground and near the toe of the slope. However, the three-dimensional model results in a higher safety factor (>1.50) than the two-dimensional equivalent section.

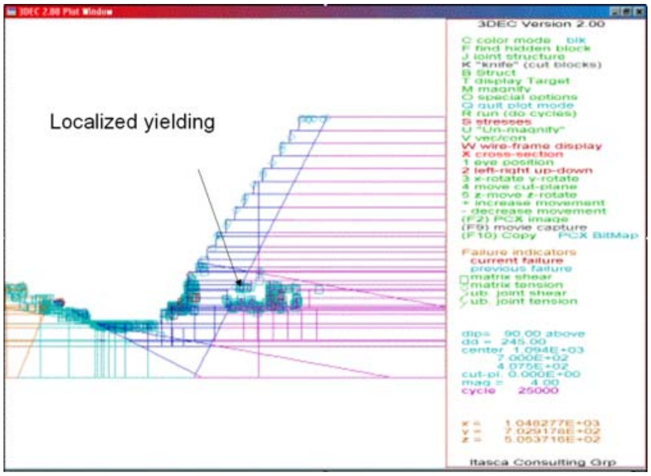

Figure 18. Vertical section through 3DEC model that is the same as the section shown in Figure 16. Only minor rock-mass failure is indicated in the three-dimensional model (FS > 1.50)

CONCLUSIONS

Two-dimensional, plane-strain finite-difference numerical models provide a greater understanding of the failure mechanisms and interaction between overall open pit slope stability and remnant underground voids. However, in practice, most remnant underground voids are not of infinite length. Two-dimensional, plane-strain numerical models have been shown to provide a conservative estimate of the overall open pit slope stability.

Until recently, three-dimensional analyses were relatively uncommon in slope design; however, recent advances in personal computers have permitted three-dimensional analyses to be performed routinely. Three-dimensional numerical models have been shown to provide a more realistic representation of the interaction between overall open pit slope stability and remnant underground voids.

ACKNOWLEDGEMENTS

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REFERENCES

Call, R D, Cicchini, P F, Ryan, T M and Barkley, R C, 2000. Managing and Analyzing Overall Pit Slopes, in *Slope Stability in Surface Mining* (Ed: W A Hustrulid, M K McCarter and D J A Van Zyl), pp 39-46 (Society for Mining and Metallurgy, and Exploration, Inc: Littleton)

Dawson, E M, Roth, W H and Drescher, A, 1999. Slope stability analysis by strength reduction. *Geotechnique*, 40(6), pp 835-840.

Dawson, E M and Roth, W H, 1999. Slope stability analysis with FLAC. FLAC and Numerical Modeling in Geomechanics, in *Proceedings of the International FLAC Symposium on Numerical Modeling in Geomechanics* (Ed: Detournay and Hart) pp 3-9 (Balkema: Rotterdam).

Itasca Consulting Group, Inc., 2000. *FLAC (Fast Lagrangian Analysis of Continua)* Version 4.0. Minneapolis: ICG

Itasca Consulting Group, Inc., 2002. FLAC^{3D} *(Fast Lagrangian Analysis of Continua in 3 Dimensions)* Version 2.1. Minneapolis: ICG

Itasca Consulting Group, Inc., 2003. *3DEC(3-Dimenional Distinct Element Code)* Version 3.0. Minneapolis: ICG

Lorig L and Varona P, 2000. Practical Slope-Stability Analysis Using Finite-Difference Codes, in *Slope Stability in Surface Mining* (Ed: W A Hustrulid, M K McCarter and D J A Van Zyl), pp 115-124 (Society for Mining and Metallurgy, and Exploration, Inc: Littleton)

Loubser, P, 1994. Geotechnical Assessment of the West Wall of KCGM's Super Pit. *Proceedings of the 4th Large Open Pit Conference*. pp 145 - 158 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Pierce, M, Brandshaug, T, Hustrulid, W and Lorig, L. 1999. Geotechnical Assessment of Ultimate and Intermediate Pit Slopes at the main Cresson Mine: Vols. I (Report) and II (Appendices). Itasca Consulting Group, Inc., report to Cripple Creek and Victor Gold Mining Company, ICG99-1032-11

Pierce, M, Brandshaug, T and Ward, M, 2000. Slope Stability Assessment at the Main Cresson Mine in *Slope Stability in Surface Mining* (Ed: W A Hustrulid, M K McCarter and D J A Van Zyl), pp 239-250 (Society for Mining and Metallurgy, and Exploration, Inc: Littleton)

Sjöberg, J, 1999. *Analysis of Large Scale Rock Slopes*, PhD thesis, Luleå University of Technology, Luleå

Walton G, and Taylor, R K, 1977 Likely constraints on the stability of excavated slopes due to underground coal workings, *Proceedings of the Conference on Rock Engineering*, Newcastle upon Tyne pp 329-349

Watters, R J, Finn, D and Coulthard, J. 1989. Pit slope instability problems induced by disused underground mine workings. *Proceedings of the 25th Symposium on Engineering Geology and Geotechnical Engineering*, pp 101 – 106 (Balkema: Rotterdam).

Watters, R J, Rehwoldt, E B and Coulthard, J, 1990. Effects of abandoned underground workings on open pit slope design. *Proceedings of the 31st US Rock Mechanics Symposium : Rock Mechanics Contributions and Challenges*, pp 721 – 728 (Balkema: Rotterdam).

Zettler, A H, Poisel, R, Roth, W and Preh, A, 1999. Slope stability analysis based on the shear reduction technique in 3D, in *Proceedings of the International FLAC Symposium on Numerical Modeling in Geomechanics* (Ed: Detournay and Hart) pp 11-16 (Balkema: Rotterdam).